The Fundamental but Misunderstood Conductivity Cell Constant *or* Why a Little Knowledge can be a Dangerous Thing

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Every day our technical support people answer questions on the selection of sensors for pH, ORP, conductivity, dissolved oxygen and turbidity. No issue generates more confusion than cell constants for conductivity sensors. The vast majority of returns we process are for conductivity sensors that were ordered with the wrong cell constant. It turns out that cell constants are something that we all read but that most of us don't really understand. Pick the wrong cell constant for a probe and your analyzer will happily give you numbers to 3 significant digits. The only problem those numbers are wrong.

I am forced to confess that, for many months, I was confused about what a cell constant is. So I thought I'd try to bring clarity to a subject that can be misconstrued in so many ways.

Before we define a cell constant let's back up and explain why we need one at all.

1. Two Ways of Saying the Same Thing

Conductivity sensors measure the propensity of an aqueous volume of water containing dissolved solids to conduct electricity. As such, water is no different than a copper wire, which conducts electricity via electrons. The main difference is that water conducts electricity through both positively charged ions ("cations") and negatively charged ions ("anions").

Though the principle of electrical conduction is the same in either case we use one physical unit when describing conduction through a wire and another when describing conduction through an aqueous sample. For the former we use units of **resistance**, i.e. ohms. If you remember your high school electronics then you surely remember the most basic of all relationships—Ohm's Law. This simple relationship states that the resistance of a material equals the voltage across that material divided by the current flowing through it:

$$R = \frac{V}{I} \qquad \text{Equation 1}$$

We could have just as easily inverted Ohm's Law, i.e. $1/R = I/V$, and used units of reciprocal ohms, or Siemens (or, if you're old enough, "mhos"). In this case we use the term **conductance** instead of resistance. The first key point is that, if we know one, we know the other. The second key point is that both resistance and conductance are properties of the **bulk** sample and do not reflect **intrinsic** properties of the sample i.e. properties that do not depend on the size or shape of the material. An analogy is that the mass or volume of an object relates to the size and shape of the object. Neither gives us any information about the nature of the material in question. However the density reflects an intrinsic property of that sample and does not depend in any way on how big that object is or how it's shaped.

We like dealing with the bulk property of resistance and its ohm (Ω) units when we are dealing with electronics because the wires are fixed in size and all we really care is how well they conduct. But, when we deal with aqueous solutions, ohms aren't very convenient. That's because our worldview is very different from those in the electronics arena. We don't care about the resistance or the conductance of water. We care about what's in the water. After all, that's what a conductivity measurement is—it's a surrogate measurement for "stuff" in the water. The concentration of dissolved solids in the water is an intrinsic property of that sample of water. (The fact that some of that "stuff" consist of organic compounds that don't dissolve into ions—like sugar—and, therefore, don't conduct electricity is an inconvenient truth that we blissfully ignore.)

If we want to use resistance or conductance measurements then we have to further define a parameter that is specific to the intrinsic properties of the solution, i.e. one that is proportional to the concentration of ions in water. We know that, for a wire, conductance is proportional to the cross section divided by the length of a wire conductor. In our mirror image world of resistance we say that resistance is inversely proportional to the length of the conductor divided by the cross section). In other words a sample of aqueous solution, or a wire, that has twice the cross section of another sample has twice the conductance. That's why we use larger diameter wire (smaller AWG values) for wires that carry a lot of current. We also understand that long wires have greater resistance than short wires.

2. Sorry but Here Comes a Little Arithmetic

We illustrate the relationship between conductance (or resistance) and dimensions in Figure 1, in which we depict a pair of rectangular electrodes between which ions in an aqueous sample flow. The distance between the electrodes is D and the area of the electrodes is A. If the electrodes were round this figure could just as well be a section of a copper wire with a cross section of A and a length of D.

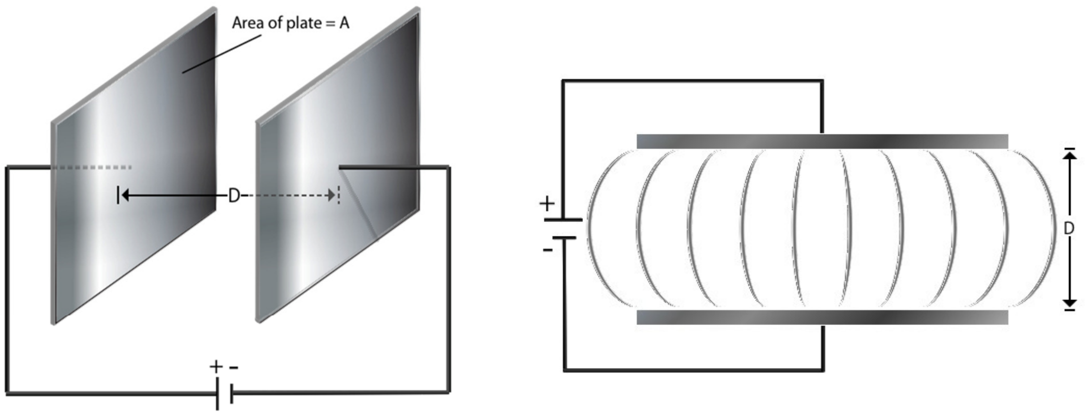

Figure 1 – On the left is a schematic of conductivity sensor based on parallel plates. The cell constant is equal to the ratio of the separation between the plates to their surface area. On the right is the view parallel to the plates showing the electric field lines. Only in the center are the lines straight so the ratio D/A is only an approximation to the cell constant (unless the plates were infinitely large!).

To turn the conductance measurement into an intrinsic property of the solution (or wire) we remove its dependence on the sample size by dividing the conductance by the surface area of the electrodes and multiplying it by the distance between electrodes (or length of wire).

$$\sigma = S\,\frac{D}{A} \qquad \text{Equation 2}$$

If we divide the conductance by the cross section of the wire we will get a parameter whose value is independent of the shape or dimensions of the wire or—in our case—the size of the aqueous sample. The conductance multiplied by the ratio D/A of the conducting sample is what we know to be **conductivity.** As the simple relationship above shows conductivity has units of Siemens/cm. To get even one little Siemen/cm in the real world of water measurement we would have to dump a lot of salt so we prefer to use units of thousandths of a Siemen/cm, or mS/cm or millionths of a Siemen/cm, or µS/cm.

We now understand that the worlds of conductance and resistance are mirror images of each other. Invert one and you get the other. So the analogue of conductivity in the resistance world is **resistivity** and has units of Ω-cm. We actually use units of resistivity, instead of conductivity, when we deal with very pure water. This is the regime where the conductivity is less than 1 µS/cm or, in resistivity terms, greater than 1 MΩ-cm. Ultra-pure water has a resistivity value of 18 MΩ-cm. This finite value for a substance that appears to be a good insulator is due to the mobile H^+ and OH^- ions—the ions that give us pH.

3. Why Being Smart is Hurting You

Figure 1does more than just explain how conductance and conductivity re related. It shows you how to build a sensor to measure conductivity and it shows you how to calculate its cell constant. You simply take two flat electrodes, apply a voltage across them and measure the current between the electrodes. The voltage is an AC one since a DC current will allow charges to build up on the electrodes and decrease the current. Two parallel plates make the calculation of the cell constant easy. We simply divide the distance between the plates by the area of either one of the plates.

There’s only one problem with it. It’s only approximately correct—it is an oversimplification of the way conduction in a solution works. See the straight lines of the electric field that run between the plates? Well they’re only straight in the middle of the plates. As one veers away from the center the field lines bow out more and more—like the field lines around a magnet. That means our simple Equation 2 is only “good enough for government work.”

This is why a little knowledge can be a dangerous thing. It’s especially true with those of us who actually understand Equation 2. We “get” that almost no one measures conductivity with parallel electrodes as depicted in Figure 1. And, even if we did, we “get” that the finite size of the electrodes makes Equation 1 only an approximation. We know that real sensors look like the ones shown in Figure 2. Nonetheless, we think that, if only we had a sophisticated enough mathematical model we can convert Equation 2 into something more realistic. We can pull out of our mathematical hat a more complex formulation of the cell constant. Any electrical engineering student with a MATLAB license can do this little trick. If you remember your calculus then you know it will involve a big, fat integral. As messy as it may be we really can calculate the cell constant for any conductivity sensor that uses conductors to measure current between then.

Figure 2 - Most contacting sensors do not have parallel plates but cylindrical electrodes that are either parallel or concentric.

But what do we do with conductivity sensors that don't measure current between electrodes? I'm thinking specifically of toroidal sensors that operate on a completely different principle. If you're like me then you think that the concept of a cell constant doesn't apply. But you'd be wrong.

To understand a cell constant we have to become dumber. We have to forget we ever saw Figure 1. We have to forget that, with our keen insight, we figured out that the cell constant is really just a geometric factor. Let's forget about all that nonsense involving electrode geometries and triple integrals. Instead let's just turn the cell constant into proportionality constant that we will simply call K so that Equation 2 becomes the even simpler Equation 3.

$$\sigma = \mathrm{K\,S} \qquad \text{Equation 3}$$

With this seismic shift in thinking the cell constant is just a number, with the familiar units of S/cm (or mS/cm or µS/cm). Its only role is to convert conductance readings values into the conductivity values we seek.

Before moving on I feel compelled to admit that I was a little loose in my ear analogy between the intrinsic property of density and that of conductivity. We multiply the density of a material—the intrinsic property—by the volume of an object to give us the total mass—the bulk property of that object. But we don't multiply the conductivity—the intrinsic property of conducting medium—by this new proportional constant to give us the conductance—the bulk property of that medium. We actually do the opposite. If we wanted to maintain the analogy we would use 1/K as our proportionality constant in the following restatement of Equation 3:

$$\mathrm{S} = \frac{1}{K}\,\sigma \qquad \text{Equation 4}$$

Nevertheless, once we start from this humble interpretation of the cell constant we can go ahead and give some meaning to the cell constant for the case of a contacting conductivity sensor. If that sensor consists of two parallel flat electrodes then Equation 2 gives us a pretty good approximation to the cell constant, i.e. K ≈ D/A. If that sensor consists of cylindrical electrodes like the ones in Figure 2 then we still can use the concept embodied in Equation 2 even if we don't actually use it calculate κ, i.e. the wider apart the electrodes the larger the cell constant and the larger the surface area of the electrodes the smaller the cell constant. We can hire an engineer to calculate the cell constant from any electrode geometry—but why bother? Let's just call the cell constant a proportionality constant and leave it at that.

4. Now Let's Have Some Fun

I didn’t write this white paper just to expound upon the philosophical underpinnings of the conductivity cell constant. Armed with the concept of a cell constant, we can calculate the cell constant of a conductivity sensor using nothing more than a conductivity analyzer and a precision resistor. Even if it isn’t a necessity it’s a fun experiment. Here’s how you do it for a contacting sensor:

1. Connect a sensor with a nominal cell constant of 1 to the analyzer. Immerse it in a calibration solution. Any value between 200 to 1000 µS/cm will work. (Remember to suspend the sensor in the middle of the solution. If it touches the sides it will partially short circuit and you will measure a lower conductivity). Calibrate the sensor to that solution. Keep in mind that you are really calibrating the analyzer to the sensor, not the other way around.
2. The sensor likely has 4 wires (5 if there is a shield wire) that connect to the analyzer. Two are from the electrodes and two are from a temperature element—usually a thermistor or RTD. Disconnect the two electrode wires from the analyzer and replace them with a precision 1kΩ resistor. A 1 kΩ resistor is equivalent to a conductance of 0.001 S—1000 µS.
3. Measure the conductivity of the resistor. Take the value you read in µS and move the decimal point 3 places to the left. For instance if you read 957 µS/cm then the cell constant of the probe is 0.957.

An alternate method is to substitute a decade box for the 1 kΩ resistor. You would then dial in the resistance required to make the analyzer read 1000 µS/cm. That resistance, divided by 1000, is the cell constant.

Since we know that resistance is the inverse of conductance we can generalize the procedure for any resistor and for any cell constant. The following simple equation will spare you the mental

$$K = 10^{-6}\,R\,(k\Omega)\,x\,S\,(\mu S) \qquad \text{Equation 5}$$

Equation 5 states that the cell constant is equal to the resistance (in kΩ) of our precision resistor multiplied by the conductivity of the calibration solution (in µS/cm) and further multiplied by a factor of 10^{-6} simply to make the units consistent.

Do you really need to do this little exercise? In most cases the answer is “not really.” When you buy a conductivity sensor it comes with a cell constant associated with it. If the true cell constant differs from the nominal constant stamped on the probe then calibration will account for the discrepancy. But if you ever have doubt as to what the cell constant of a probe is this simple, fast and accurate experiment will give you your answer. And, even if you’re only 10% geek, it’s a lot of fun.